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A DYNAMIC STIFFNESS FORMULATION FOR THE ANALYSIS OF SECONDARY SYSTEMS

DIONISIO BERNAL*†
Northeastern University, Boston MA 02115, U.S.A.

SUMMARY

Formulation of a frequency-domain substructure approach for the analysis of secondary systems is presented. The total system contemplated includes the primary structure, the secondary system, and the foundation medium, which is also treated as a substructure. A dynamic stiffness matrix in physical co-ordinates characterizes each one of the substructures. Elimination of the internal degrees of freedom of the primary structure prior to assembly of the equations for the coupled system is carried out with the aid of a truncated set of unconstrained normal modes. Accounting for the residual static flexibility of the truncated modes obviates potential problems of rank deficiency resulting from modal truncation. The formulation contemplates an arbitrary multi-component scattered motion at the soil–structure interface and imposes no limitations on the configuration of the primary or the secondary system. Connectivity between the systems is treated as an arbitrary linear relation between selected co-ordinates in each substructure. This feature is shown to be useful for modelling the commonly encountered situation where secondary systems are attached to torsionally eccentric structures.

KEY WORDS: substructures; frequency domain; secondary systems

1. INTRODUCTION

During seismic excitation the motion at the supports of secondary (S) systems can be viewed, on the assumption of linear behaviour, as the sum of two contributions: (1) the motion at the attachment points due to seismic excitation on the primary (P) system and (2) the additional motion from the reactions of the S system on the P system. The simplest approach to arrive at a decoupled analysis is to neglect the contribution to support motion that derives from feedback. The foregoing simplification has been widely used in practice and forms the basis of the traditional Instructure Response Spectrum (IRS) approach.[1–3] Needless to say, decoupling is highly desirable since responsibility for the design of the P and the S systems typically resides in different engineering groups, or is carried out at different times. Moreover, since the design of the P system is not usually affected by the feedback reactions, decoupling eliminates the need to consider the large coupled P–S model when a number of alternatives are contemplated during the design of the S system.

* Correspondence to: Dionisio Bernal, Department of Civil and Environmental Engineering, 427 Snell Engineering Center, Northeastern University, Boston, MA 02115, USA.
† Associate Professor, Civil and Environmental Engineering.

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The error incurred from neglecting feedback is small when the S system is light and its frequencies are not near any of the frequencies of the important modes of the P system. In many practical cases, however, conditions are such that P–S interaction must be considered if reasonably accurate predictions of the response of the S system are to be obtained.[4–8] An accurate approach that retains some of the advantages of a decoupled solution is to synthesize the modal characteristics of the coupled system from those of the P and S components.[8,9] A desirable feature of this technique is the fact that the resulting system can be analyzed for the response spectrum specified at the base of the P system, obviating complications when the S system is multiply supported. Detracting from the usual simplicity of a response spectrum solution, however, are the complex-valued eigenproperties needed to consider the non-classical distribution of damping in the P–S system, and the fact that the correlation between tuned complex modes demands special treatment. An alternative, whereby the S system is initially idealized as a Single-Degree-Of-Freedom (SDOF) oscillator, allows presentation of the results in terms of feedback modified response spectra.[6,8,10] Use of these spectra to compute the response of multi-degree-of-freedom S systems is approximate. Recent work on P–S systems using frequency-domain techniques include the development of transfer functions for SDOF secondary systems developed by Gupta[11] and the extension of these transfer functions to multiply supported S systems supported on fixed base buildings by Dey and Gupta.[12] Extensive discussion of most of the techniques that have been developed for the analysis of P–S systems can be found in state of the art reviews by Chen and Soong,[13] Singh[14] and Soong.[15] Examination of the state of the art from a more general perspective, including a discussion on code provisions, has been recently presented by Villaverde.[16]

A situation not usually contemplated in the techniques for the analysis of P–S systems is that where the P system interacts with a compliant foundation. A notable exception is a recent paper by Dey and Gupta[17] where the primary system is a 2-D building with a rigid base supported on a compliant soil and subjected to a translational excitation. For more general cases one can, in principle, proceed by treating the P–S system as a unit and perform the analysis using any of the techniques that have been developed for soil–structure systems.[18–21] An option that avoids consideration of the coupled system as a unit is to calculate the complex-valued eigenproperties of the flexibly supported P system and then use the general approach presented by Suarez and Singh[22] to synthesize the modal characteristics of the P–S system from that of the components. This option, however, is hampered by the fact that the eigenvalue problem for the P system on compliant foundation is non-linear and must be solved iteratively (unless the impedance functions are approximated using a lumped parameter model[23]).

A formulation of general applicability that does not operate with the complex eigensolution of the flexibly supported P–S system is presented in the following sections. The technique is a frequency-domain substructure approach in physical co-ordinates belonging to the methodology known as frequency-domain structural synthesis.[24,25] Substructures P, S and G, where the G substructure represents the foundation impedance are considered. Economy is attained by eliminating all of the non-connecting co-ordinates of the P substructure (typically a large fraction of the total number) prior to assembly of the system equations. Elimination of the unwanted co-ordinates by static condensation, although exact in the frequency domain, is argued as impractical since the size of the matrix that needs to be inverted (at each one of the frequencies considered in the analysis) equals the number of DOF to be removed. The alternative selected computes the dynamic flexibility of the P system using a truncated set of unconstrained modes and then obtains the desired dynamic stiffness by inversion of the appropriate partition. Potential

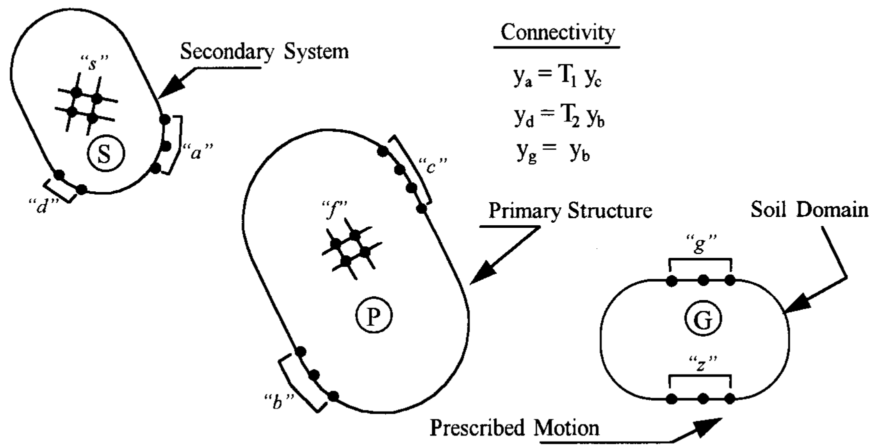

Figure 1. Substructures used in the formulation

problems of rank deficiency resulting from modal truncation are obviated by accounting for the residual flexibility of truncated modes using an approach introduced by McNeal[26] and recently reviewed by Spanos and Majed[27] and Majed and Spanos.[28] A noteworthy feature of the formulation is the fact that the P–S connectivity is treated as an arbitrary linear relationship between certain sets of co-ordinates. This feature is shown useful when modeling the commonly encountered situation of secondary systems that interact with torsionally eccentric primary structures. The formulation is compatible with specification of the seismic input as an arbitrary multi-component excitation and, except for approximation resulting from modal truncation in the representation of the P substructure, is exact.

2. FORMULATION

Figure 1 depicts the substructures considered, substructure S represents the secondary system, P the primary structure and G the soil domain. The DOF in substructure S are partitioned into sets s, a and d and those in P into c, f and b. For the S substructure the DOF in s are internal, those in a are 'attached' to the P substructure and the d-set contains DOF with connectivity to the foundation. It is worth noting that the d-set, due to kinematic constraints in the P system, may not be null even in the absence of a physical connection between S and G, as in example #2. For substructure P, set-c is 'connected' to S while sets b and f contain the DOF at the soil–structure interface and the internal DOF that are to be eliminated, respectively. The soil domain is assigned DOF g at the soil–structure interface and a set z, which is used to introduce the prescribed scattered motion[‡].

The reaction forces at the soil–structure interface, $\mathbf{L}_g$, are related to the difference between the motions at the z and the g nodes by the dynamic stiffness matrix of the soil domain $\mathbf{G}_g$,

[‡] Motion at the soil–structure interface computed in the absence of the structure but with due consideration for the excavation (if any)

namely

$$\mathbf{G}_g(\mathbf{y}_g - \mathbf{y}_z) = -\mathbf{L}_g \tag{1}$$

The dynamic stiffness equations for the S and the P substructures can be written symbolically as

$$\mathbf{S}_s\boldsymbol{\delta}_s = \mathbf{L}_s \tag{2}$$

and

$$\tilde{\mathbf{S}}_p\boldsymbol{\delta}_p = \mathbf{L}_p \tag{3}$$

where $\boldsymbol{\delta}_s = [\mathbf{y}_s\ \mathbf{y}_a\ \mathbf{y}_d]^{\mathrm{T}}$ and $\boldsymbol{\delta}_p = [\mathbf{y}_c\ \mathbf{y}_b]^{\mathrm{T}}$. The tilde in equation (3) is introduced to bring attention to the fact that in this expression the f DOF have been condensed.

Connectivity: The connectivity between the substructures is defined by the following linear transformations:

$$\mathbf{y}_a = \mathbf{T}_1\mathbf{y}_c \tag{4a}$$

$$\mathbf{y}_d = \mathbf{T}_2\mathbf{y}_b \tag{4b}$$

$$\mathbf{y}_g = \mathbf{y}_b \tag{4c}$$

Note that the equality between $\mathbf{y}_g$ and $\mathbf{y}_b$ is introduced without loss of generality.

Dynamic stiffness for the coupled system: Since the dynamic stiffness expressions are given in terms of physical co-ordinates the assembly process is straightforward. In particular, the dynamic stiffness relations are transformed to common co-ordinates and the results are simply added. The common co-ordinates are $[\mathbf{y}_s\ \mathbf{y}_c\ \mathbf{y}_b]^{\mathrm{T}}$. After simple transformations and additions one gets

$$\begin{bmatrix} S_{ss} & S_{sa}T_1 & S_{sd}T_2 \\ & T_1^{\mathrm{T}}S_{aa}T_1 + \tilde{S}_{cc} & T_1^{\mathrm{T}}S_{ad}T_2 + \tilde{S}_{cb} \\ & \text{sym.} & T_2^{\mathrm{T}}S_{dd}T_2 + \tilde{S}_{bb} + G_g \end{bmatrix} \begin{Bmatrix} y_s(\omega) \\ y_c(\omega) \\ y_b(\omega) \end{Bmatrix} = \begin{Bmatrix} L_s(\omega) \\ 0 \\ G_g y_z(\omega) \end{Bmatrix} \tag{5}$$

where the argument ω has been added to emphasize that the expression applies in the frequency domain. Note that the relationship in equation (5) also holds between the nth derivative of $[\mathbf{y}_s\ \mathbf{y}_c\ \mathbf{y}_b]^{\mathrm{T}}$ and the nth derivative of $\mathbf{y}_z$ (provided $\mathbf{L}_s = \mathbf{0}$). It is opportune to note that because of the large difference in the magnitudes of the properties of the P and the S systems, the condition number for the coefficient matrix in equation (5) may be high in relation to the physical sensitivity of the solution to the parameters of the problem. While condition number difficulties were not encountered in the numerical examples considered, the condition number for poorly scaled problems can, if necessary, be reduced by appropriate scaling.[29]

2.1. Transfer functions

The transfer functions for each one of the seismic inputs can be readily obtained from equation (5) by taking $\mathbf{L}_s = \mathbf{0}$. In particular, considering a temporal excitation $x_g(t)$ with a spatial distribution $\mathbf{q}$ one can write

$$\mathbf{y}_z(\omega) = \mathbf{q}x_g(\omega) \tag{6}$$

Substituting equation (6) into equation (5), taking $x_g(\omega) = 1$, and inverting one gets

$$\begin{Bmatrix} \mathscr{H}_s(\omega) \\ \mathscr{H}_c(\omega) \\ \mathscr{H}_b(\omega) \end{Bmatrix} = \begin{bmatrix} S_{ss} & S_{sa}T_1 & S_{sd}T_2 \\ & T_1^{\mathrm{T}}S_{aa}T_1 + \tilde{S}_{cc} & T_1^{\mathrm{T}}S_{ad}T_2 + \tilde{S}_{cb} \\ & \text{sym.} & T_2^{\mathrm{T}}S_{dd}T_2 + \tilde{S}_{bb} + G_g \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ G_g q \end{Bmatrix} \tag{7}$$

As is evident, the transfer functions in equation (7) also apply from the nth derivative of ground displacement to the nth derivative of absolute structural response.

2.2. Case without SSI

While rigid soil conditions can be enforced by using large values for $\mathbf{G}_g$, specific expressions derived explicitly for this condition are convenient. The solution follows by noting that as $\mathbf{G}_g \to \infty$ $\mathbf{y}_b \to \mathbf{y}_z$, from an inspection of equation (5) one gets

$$\begin{Bmatrix} y_s \\ y_c \end{Bmatrix} = [A(\omega)]^{-1}\left[\begin{Bmatrix} L_s \\ 0 \end{Bmatrix} - [B(\omega)]\{y_z\}\right] \tag{8}$$

where

$$[A(\omega) = \begin{bmatrix} S_{ss} & S_{sa}T_1 \\ \text{sym} & T_1^{\mathrm{T}}S_{aa}T_1 + \tilde{S}_{cc} \end{bmatrix} \tag{9a}$$

and

$$[B(\omega)] = \begin{bmatrix} S_{sd}T_2 \\ T_1^{\mathrm{T}}S_{ad}T_2 + \tilde{S}_{cb} \end{bmatrix} \tag{9b}$$

2.3. Solution for various types of input

For a deterministic time history (or histories) the response can be readily obtained from equation (5) by inverse Fourier transformation. When the excitation is prescribed in terms of PSD one obtains the spectral density of the response, under the assumption of stationarity, as the square of the modulus of the appropriate transfer function times the PSD of the input. The maximum response can then be computed from the spectral density of the output and an assumed effective duration using well-established procedures.[30] If the assumption of stationarity cannot be justified an evolutionary spectral approach can be used.[30–32] In this case one needs to compute a time-dependent transfer function which is defined as the Fourier Transform of a modulated version of the impulse response for the desired quantity, namely

$$[H_j(\omega, t)] = \int_{-\infty}^{\infty} h_j(\tau)\cdot c(t-\tau)\cdot \mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau \tag{10}$$

where $c(t)$ is a deterministic time modulation and $\hbar_j(t)$ is given by

$$h_j(t) = \int_{-\infty}^{\infty} \mathscr{H}_j(\omega)\cdot \mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\omega \tag{11}$$

As can be seen, the essential information required is still the steady-state transfer functions in equation (7).

While use of a Response Spectrum (RS) to define the seismic input is not directly compatible with a frequency-domain formulation, conversion of the RS to a compatible PSD can be carried out and the problem solved as outlined previously. The connection between RS and PSD has been discussed by Kaul[33] and Vanmarcke,[30] among others. Key to the practicality of the preceding formulation is the reduction in problem size realized by describing the P substructure in terms of its dynamic stiffness for the connecting co-ordinates. Methodology for the efficient calculation of this dynamic stiffness is presented in the following section.

3. DYNAMIC STIFFNESS FOR A SUBSTRUCTURE

The equation of motion for a linear substructure can be written in matrix form in the frequency domain as

$$\mathbf{S}\boldsymbol{\delta} = \mathbf{F} \tag{12}$$

where the dynamic stiffness $\mathbf{S}$, for a system with viscous damping, is given by

$$\mathbf{S} = \mathbf{K} - \omega^2\mathbf{M} + \mathbf{C}\omega\mathrm{i} \tag{13}$$

In equation (12) the vector of physical co-ordinates $\boldsymbol{\delta}$ is assumed to be ordered $\boldsymbol{\delta} = [\boldsymbol{\delta}_{\mathrm{m}}\ \boldsymbol{\delta}_{\mathrm{sl}}]^{\mathrm{T}}$, where $\boldsymbol{\delta}_{\mathrm{m}}$ and $\boldsymbol{\delta}_{\mathrm{sl}}$ are the partitions containing the co-ordinates to be retained (masters) and to be removed (slaves). The load vector is consistently partitioned as $\mathbf{F} = [\mathbf{F}_{\mathrm{m}}\ \mathbf{0}]^{\mathrm{T}}$, where the condition of no external dynamic loads at the slave co-ordinates has been introduced. The expression in the frequency-domain relating loads and displacements at the master co-ordinates is

$$\tilde{\mathbf{S}}_{\mathrm{m}}\boldsymbol{\delta}_{\mathrm{m}} = \mathbf{F}_{\mathrm{m}} \tag{14}$$

Evaluation of the dynamic stiffness for the master co-ordinates $\tilde{\mathbf{S}}_{\mathrm{m}}$by direct condensation of equation (12) is inefficient.[34] This contention is made evident by noting that the approach involves the solution of simultaneous equations equal in number to the DOF to be condensed and that the set of equations (barring introduction of some interpolation scheme) must be solved at each one of the frequencies considered in the analysis. An efficient alternative is to form the dynamic flexibility using the undamped eigensolution and then to compute the dynamic stiffness for the master co-ordinates by inversion of the appropriate partition. Specifically, solving the eigenvalue problem for matrices $\mathbf{K}$ and $\mathbf{M}$ and assuming classical damping one can express equation (13) as

$$\mathbf{S} = (\boldsymbol{\Phi}^{\mathrm{T}})^{-1}\mathbf{D}\,\boldsymbol{\Phi}^{-1} \tag{15}$$

where $\boldsymbol{\Phi}$ is the mass normalized modal matrix and $\mathbf{D}$ is a diagonal matrix with the jth element given by

$$d_j = \omega_j^2 - \omega^2 + 2\omega_j\omega\xi_j\mathrm{i} \tag{16}$$

where ω_j and ξ_j are the natural frequency and the damping ratio of the jth mode. Multiplying equation (12) by $\mathbf{H} = \mathbf{S}^{-1}$ and introducing the standard partitioned notation one gets

$$\boldsymbol{\delta}_{\mathbf{m}} = \mathbf{H}_{\mathbf{mm}}\mathbf{F}_{\mathbf{m}} \tag{17}$$

From inspection of equation (15) and the definition of $\mathbf{H}$ one concludes that

$$\mathbf{H}_{\mathbf{mm}} = \mathbf{\Phi}_{\mathbf{m}}\mathbf{D}^{-1}\mathbf{\Phi}_{\mathbf{m}}^{\mathrm{T}} \tag{18}$$

where $\mathbf{\Phi}_{\mathbf{m}}$ is the partition of the modal shape matrix associated with the *master* co-ordinates. From equations (14), (17) and (18) the dynamic stiffness is given by

$$\tilde{\mathbf{S}}_{\mathrm{m}} = (\mathbf{\Phi}_{\mathbf{m}}\mathbf{D}^{-1}\mathbf{\Phi}_{\mathbf{m}}^{\mathrm{T}})^{-1} \tag{19}$$

Use of equation (19) is generally more efficient than a direct condensation of equation (12) because the gains from the smaller size matrix that needs to be inverted are realized at each one of the frequencies in the analysis and the eigenvalue problem needs to be solved only once. It is finally worth noting that when the substructure considered is unconstrained one must ensure that the rigid body modes in $\mathbf{\Phi}$, which are not unique, are selected to be orthogonal among themselves.

3.1. Correction for truncated modes: residual flexibility

Computational efficiency suggests that the evaluation of the dynamic flexibility with equation (18) be done using a truncated modal basis. The truncated basis, however, may lead to a singular $\mathbf{H}_{\mathbf{mm}}$ making it impossible to carry out the inversion necessary to obtain the desired stiffness matrix. While the number of modes needed to ensure that equation (18) is not rank deficient is at least equal to the number of master co-ordinates, the actual number of modes to ensure full rank depends on the details of the problem. As described in this section, rank related restrictions on the number of modes can be eliminated by adding the residual static flexibility of the truncated modes to the result from equation (18). It should be noted that the preceding issue is separate from the more traditional question pertaining to the number of modes needed to attain a certain level of accuracy. The question of accuracy is briefly touched on in connection with the results presented in Figure 6.

When the substructure considered is constrained, the residual flexibility can be evaluated by subtracting the result from equation (18), at $\omega = 0$, from the total static flexibility. For an unconstrained substructure, however, the flexibility at $\omega = 0$ is not defined and the elementary procedure must be modified. The modified approach operates by computing the fraction of the flexibility that is not associated with rigid body modes. In particular, the dynamic flexibility can be expressed as the sum of three parts, namely

$$\mathbf{H}_{\mathbf{m.m}} = \mathbf{H}_{\mathbf{mm}}^{\mathbf{rb}} + \mathbf{H}_{\mathbf{mm}}^{\mathbf{rm}} + \mathbf{H}_{\mathbf{mm}}^{\mathbf{tr}} \tag{20}$$

where the superscripts, rb, rm and tr stand for rigid body modes, retained modes and truncated modes. While the total flexibility is not defined at $\omega = 0$, the last two terms on the right side of equation (20) can be evaluated without difficulty. Removing the flexibility associated with the rigid body modes one can write

$$(\mathbf{H}_{\mathbf{mm}}^{\mathbf{rm}} + \mathbf{H}_{\mathbf{mm}}^{\mathbf{tr}})\mathbf{F}_{\mathbf{m}} = \boldsymbol{\delta}_{\mathrm{r}} \tag{21}$$

where the displacement vector $\boldsymbol{\delta}_{\mathrm{r}}$ is orthogonal to the rigid body modes (since there is no associated flexibility). Assume that $\mathbf{G}_{\mathbf{0}}$ is a flexibility matrix computed by introducing a set of statically determinate constraints. Provided the applied load vector is orthogonal to the vector space defined by $\mathbf{M\Phi}^{\mathrm{rb}}$ the reactions in the constraints will be zero, and one can write

$$\mathbf{G}_0\mathbf{A}\mathbf{F}_{\mathrm{m}} = \boldsymbol{\delta}_{\mathrm{r}} + \mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}}\mathbf{Y} \tag{22}$$

where the sweeping matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \mathbf{I} - \mathbf{M}\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}}(\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}})^{\mathrm{T}} \tag{23}$$

and $\mathbf{Y}$ contains amplitudes of rigid body motion which depend on the location of the statically determinate constraints. Substituting equation (21) into (22), premultiplying by $(\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}})^{\mathrm{T}}\mathbf{M}$ and recognizing that $(\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}})^{\mathrm{T}}\mathbf{M}\boldsymbol{\delta}_{\mathbf{r}} = \mathbf{0}$ and $(\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}})^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}} = \mathbf{I}$, one gets

$$\mathbf{Y} = (\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rb}})^{\mathrm{T}}\mathbf{M}\mathbf{G}_0\mathbf{A}\mathbf{F}_{\mathbf{m}} \tag{24}$$

Combining equations (21), (22) and (24) the flexibility of the truncated modes can be written in terms of the total static flexibility minus the flexibility of the retained modes, namely

$$\mathbf{H}_{\mathbf{m}}^{\mathbf{tr}} = \mathbf{A}^{\mathrm{T}}\mathbf{G}_0\mathbf{A} - \mathbf{H}_{\mathbf{mm}}^{\mathbf{rm}} \tag{25}$$

or

$$\mathbf{H}_{\mathbf{m}}^{\mathbf{tr}} = \mathbf{A}^{\mathrm{T}}\mathbf{G}_0\mathbf{A} - \mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rm}}(\mathbf{D}^{-1})_{\omega=0}\mathbf{\Phi}_{\mathbf{m}}^{\mathbf{rm}} \tag{26}$$

The dynamic flexibility, with due consideration for the residual static flexibility of the truncated modes is, therefore

$$\mathbf{H}_{\mathbf{mm}} = \mathbf{\Phi}_{\mathbf{m}}\mathbf{D}^{-1}\mathbf{\Phi}_{\mathbf{m}}^{\mathrm{T}} + \mathbf{H}_{\mathbf{m}}^{\mathbf{tr}} \tag{27}$$

While equation (27) cannot be evaluated at $\omega = 0$ for an unconstrained substructure, its inverse (the result needed) is simply the static stiffness condensed to the $\boldsymbol{\delta}_{\mathbf{m}}$ co-ordinates.

4. NUMERICAL EXAMPLES

Example 1. *A secondary system idealized as a beam element with two equally spaced lumped masses is pinned to the second and the third floor levels of a three storey torsionally eccentric building as shown in Figure* 2. *The masses of the* S *system are taken as* 1/15 *and* 2/15 *of the mass of a floor. The influence of interaction and the effect of the relative location of the* S *system on the floor plan are examined by inspecting the modulus of the transfer function between base acceleration in the* N–S *direction to moment at mid-height of the* S *system. Mass and stiffness values for the building are selected so that the first fundamental mode in its fixed-base condition* (*which involves* N–S *translation and twisting*) *is* 1.0 *sec. Interaction with the* S *system is promoted by fixing the fundamental period of the* S *system at* 1.0 *sec also. Damping is taken as* 5 *and* 2 *per cent for the* P *and the* S *systems, respectively*.

Case 1. S system is attached to the west edge. From the sign convention in Figure 3 one has

$$[T_1] = \begin{bmatrix} 0 & 1 & 12{\cdot}2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 12{\cdot}2 \end{bmatrix}(\mathrm{m})$$

Case 2. S system is attached to the east edge. In this case the connectivity matrix is

$$[T_1] = \begin{bmatrix} 0 & 1 & -12{\cdot}2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -12{\cdot}2 \end{bmatrix}(\mathrm{m})$$

In both cases $[T_2]$ is null since there are no d DOF.

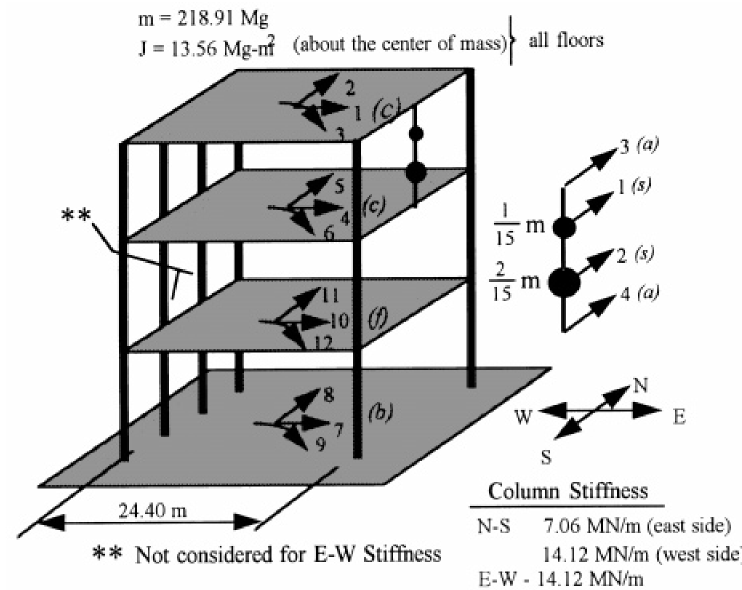

Figure 2. Example 1

The transfer function modulus for the two locations considered are depicted in Figure 3 for analyses with and without interaction. As one anticipates, the results show that the effect of feedback for this heavy tuned S system is critical. Specifically, while the maximum amplification with respect to the solution at $\omega = 0$ is over 600 for the east-side attachment when no interaction is considered, the value computed when interaction is appropriately contemplated is only around 50. From the point of view of how torsion in the P system affects the response of the S system the effect of feedback is also fundamental. For example, the area under the transfer function for the east (flexible side) attachment when no feedback is considered is 3·7 times that for attachment on the west side. It follows, therefore, that for excitations with a wide band PSD the expected response for the east side attachment (neglecting peak factor differences) is anticipated as 3·7 times larger than that for attachment to the west side. However, the ratio between the corresponding areas when interaction is considered is only 1·35, indicating that torsion in the P system actually plays a modest role in the expected response of this heavy S system. It is appropriate to note that since the results without interaction are approached as the mass and stiffness of the S system decrease proportionally, the location in plan could prove decisive for a sufficiently light system.

It is interesting to note that the clear distinction in the first resonant frequency for the two locations considered in Figure 3 is consistent with the 'stiff side' and 'flexible side' terminology widely used in the examination of building torsional response. In the current example the dynamic stiffness for the P system has been computed including all the modes in equation (27). A brief examination of the effect of modal truncation is presented in Example 2.

Example 2. *The structure considered is an* 8-*storey* 2-*D shear building supported at the surface of a soft soil layer underlain by rock. As shown in Figure* 4, *the secondary system is also idealized as*

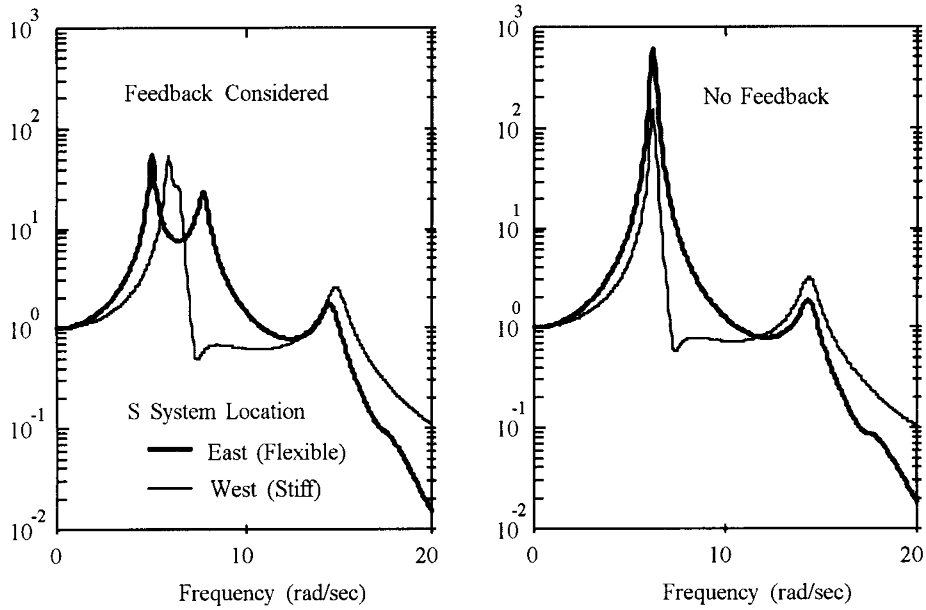

Figure 3. Normalized transfer functions between ground acceleration in the N–S direction and moment at midheight of the S system.

a shear frame with legs connected at levels 4 *and* 5. *The dynamic stiffness of the foundation is computed using the model described by Wolf.*[35] *As is evident from the results depicted in the insert, the dynamic stiffness of this shallow layer displays a strong frequency dependency. The masses and stiffness for the* P *system are chosen so the fundamental period for the fixed-base condition is* 1.0 *sec. The fundamental period of the* S *system is also chosen as* 1 *s and the masses are taken as* 1 *per cent of the mass of a floor. Rotational inertia is considered for the floor slabs but neglected for the masses of the* S *system. Consistent with the assumption of rigid girders, the vertical DOF at the connections are omitted. Damping is* 5 *and* 2 *per cent for the* P *and the* S *systems, respectively. The natural frequencies for the vibration modes of the* P *system on its unconstrained condition are*: 15·52, 21·65, 32·47, 42·30, 51·10, 58·31, 63·68 *and* 66·98 *rad/sec. The frequencies for the* S *system in its fixed base condition are*: 6·28 *and* 17·72 *rad/sec.*

Since the columns of the building are assumed inextensible, the rotation at the base is transmitted to all the floor slabs unchanged. With reference to the DOF in Figure 4 the connectivity matrices are

$$[T_1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Figure 5 shows the modulus of the transfer function between base acceleration and shear in column ‘ab’ of the S system for two values of the shear wave velocity of the soil. While not visually

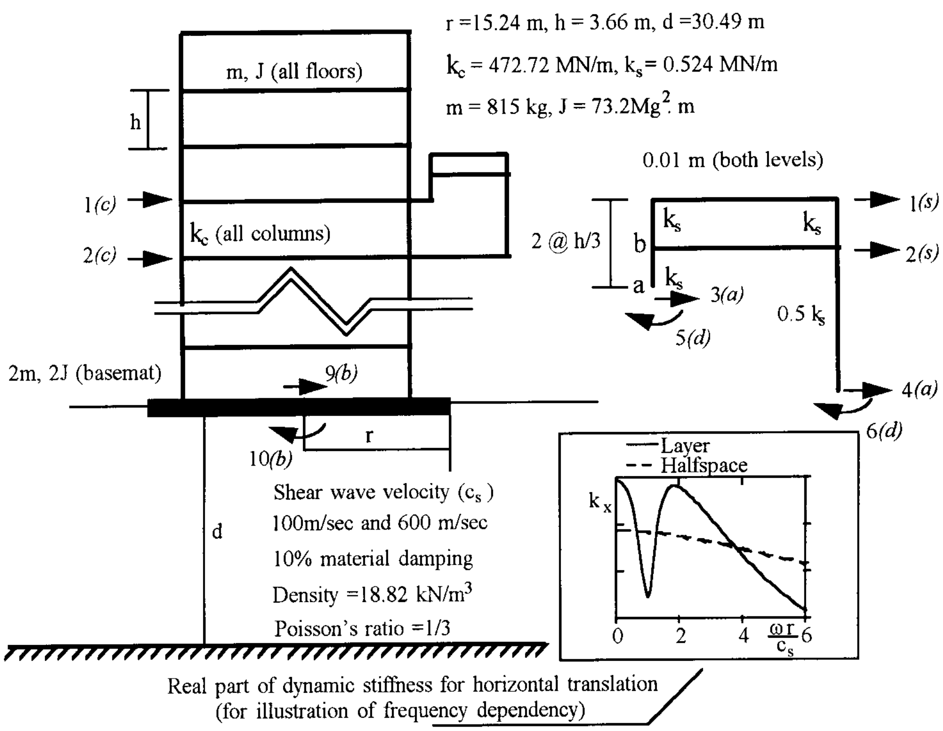

Figure 4. Example 2

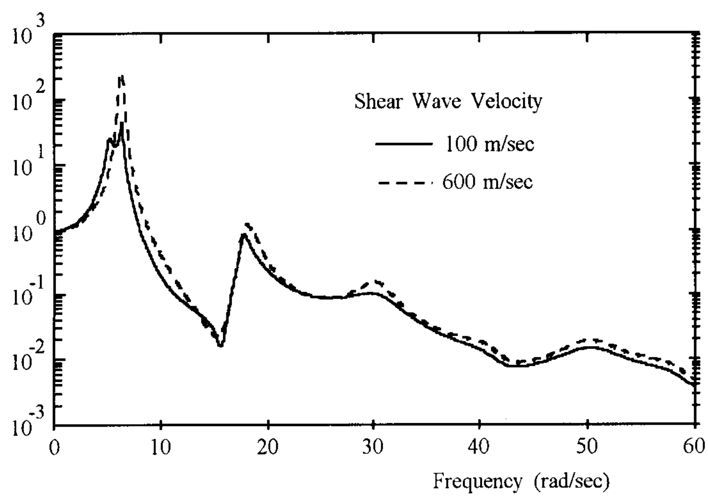

Figure 5. Normalized transfer function between base acceleration and shear in column 'ab' in the S system for Example 2 (the curves are normalized with respect to the shear at $\omega = 0$ when the shear wave velocity is 600 m/sec)

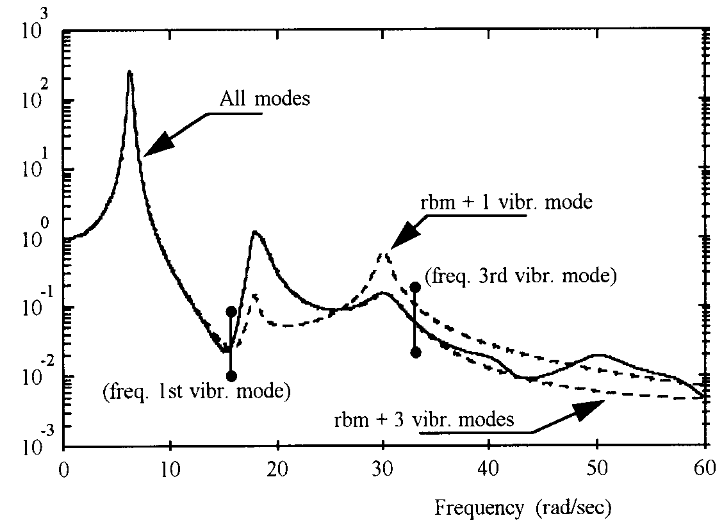

Figure 6. Effect of modal truncation in the representation of the P system on the transfer function from Fig. 5 (shear wave velocity = 600 m/sec)

striking on the logarithmic scale, the values for the softer soil ($c_s = 100$ m/sec) are significantly smaller than for the more competent one ($c_s = 600$ m/sec). In particular, numerical integration shows that the area under the transfer function for the stiff soil is 2·29 times that for the soft soil.

The influence of modal truncation on the computed results is examined in Figure 6. In particular, results computed including one and three vibration modes (in addition to the rigid body modes) are compared to the exact answer. As can be seen, the results are highly accurate up to, at least, the frequency of the last mode considered.

5. CONCLUDING REMARKS

A substructure approach for computation of the response of secondary systems attached to primary structures is presented. The formulation contemplates an arbitrary multi-component scattered motion at the soil–structure interface and imposes no limitations on the configuration of the primary or the secondary system. Since the focus is on computing the response of the S system, the DOF of the P system which do not enter explicitly in this evaluation are eliminated prior to assembly of the equations for the coupled system. Elimination of the unwanted co-ordinates is carried out efficiently with the aid of a truncated set of unconstrained modes. Potential problems of rank deficiency associated with modal truncation are obviated by incorporating the residual flexibility of the truncated modes. Kinematic constraints introduced in the definition of the models are easily accommodated by treating P–S connectivity as an arbitrary linear relation between selected co-ordinates in each substructure. This feature is shown useful for modelling the commonly encountered situation where secondary systems are attached to torsionally eccentric structures. Except for approximation resulting from modal truncation in the description of the P system, the formulation is exact and thus not limited to light secondary systems.

The approach presented retains the advantage of a decoupled analysis in the sense that, for a given connectivity, arbitrary changes in the properties of the S system can be accommodated without revisiting the P system or the foundation compliance. It is opportune to note that the possibility of using fixed-base modes instead of unconstrained modes to describe the P system was contemplated. In this option, however, the dynamic stiffness associated with the master co-ordinates cannot be expressed solely in terms of the eigensolution. The decision to use unconstrained modes was taken to avoid mixing modal and physical parameters in equation (19). The versatility of the formulation presented is demonstrated by two numerical examples. The results from Example 1 suggest that the relative position of the S system within a particular floor can play an important role if the building undergoes torsional response. This aspect of secondary system performance appears to have received little attention so far.[36]

REFERENCES

1. J. M. Biggs and J. M. Roesset, 'Seismic analysis of equipment mounted on a massive structure', in R. J. Hansen (ed.), *Seismic Design for Nuclear Power Plants*, MIT Press, Cambridge, MA, 1970, pp. 319–343.
2. M. Amin, W. J. Hall, N. M. Newmark and R. P. Kawasara, 'Earthquake response of multiply connected light secondary systems by spectrum methods', Proc., ASM E 1st Natl. Congr. on Press. Vessels and Piping, ASME, New York, 1971, pp. 103–129.
3. E. H. Vanmarcke, 'A simple procedure for predicting amplified response spectra and equipment response', Proc. 6th world Conf. on Earth. Engrg. III, Sarita Prakashan, Meerut, India, 1977, pp. 3323–3327.
4. Y. K. Wang, M. Subudhi and P. Bezler, 'Comparison study of time history and response spectrum – response for multiply supported piping systems', *Trans. 7th Int. Conf. Struct. Mech. Reactor Techn.* Chicago, Illinois **K(a)**, 1983, pp. 477–483.
5. A. K. Gupta and J. W. Jaw, 'Coupled response spectrum analysis of secondary systems using uncoupled modal properties', *Nucl. Engng. Des.* **92,** 61–68 (1986).
6. T. Igusa and A. Der Kiureghian, 'Generation of floor response spectra including oscillator–structure interaction', *Earthquake Engng. Struct. Dyn.* **13,** 661–676 (1985).
7. M. P. Singh and L. E. Suarez, 'Seismic response analysis of structure-equipment systems with non-classical damping effects', *Earthquake Engng. Struct. Dyn.* **15,** 871–886 (1987).
8. A. Asfura and A. Der Kiureghian, 'Floor response spectrum method for seismic analysis of multiply supported secondary systems', *Earthquake Engng. Struct. Dyn.* **14,** 245–265 (1986).
9. L. E. Suarez and M. P. Singh, 'Exact modal synthesis methods for seismic analysis of primary and multiply supported secondary systems', *Report No. VPI-E-86-16*, Virginia Polytechnique Institute and State University, Blacksburg, Va, 1986.
10. R. A. Burdisso and M. P. Singh, 'Seismic analysis of multiply supported secondary systems with dynamic interaction effects', *Earthquake Engng. Struct. Dyn.* 15, 1005–1022 (1987).
11. V. K. Gupta, 'Acceleration transfer function of secondary systems', *J. Engrg. Mech. ASCE*, **123**(7), 678–685 (1997).
12. A. Dey and V. K. Gupta, 'Response of multiply supported secondary systems to earthquakes in frequency domain, *Earthquake Engng. Struct. Dyn.* **27,** 187–201 (1998).
13. Y. Chen and T. T. Soong, 'State-of-the-art review: seismic response of secondary systems', *Engng. Struct.*, **10**(4), 218–228 (1988).
14. M. P. Singh, 'An overview of techniques for analysis of non-structural components', *Proc., ATC-29 Seminar and Workshop on Seismic Design and Performance of Equipment and Non-Structural Components in Buildings and Industrial Structures*, Applied tech. coun., Redwood city, Calif., 1990, pp. 215–224.
15. T. T. Soong, 'Seismic behavior of nonstructural elements – state-of-the-art report', *Proc., 10th Eur. Conf. on Earth. Engng*, Vol. 3, A.A. Balkema, Rotterdam, The Netherlands, 1994, pp. 1599–1606.
16. R. Villaverde, 'Seismic design of secondary structures: state of the art', *J. Struct. Engng. ASCE*, **23** (8), 1011–1019 (1998).
17. A. Dey and V. K. Gupta, 'Stochastic seismic response of multiply-supported secondary systems in flexible-base structures', *Earthquake Engng. Struct. Dyn.* **28,** 351–369 (1999).
18. J. P. Wolf, *Dynamic Soil–Structure Interaction*, Prentice-Hall, Englewood Cliffs, NJ, 1985.
19. J. P. Wolf, *Soil–Structure-Interaction Analysis in Time Domain*, Prentice-Hall, Englewood Cliffs, NJ, 1988.
20. J. D. Kawamoto, 'Solution of nonlinear dynamic structural system by a Hybrid Frequency–Time-domain approach', *Research Report R 83-5*, Massachusetts Institute of Technology, Department of Civil Engineering, Cambridge, MA 1983.

21. D. Bernal and A. Youssef, 'A hybrid time–frequency domain formulation for non-linear soil-structure interaction', *Earthquake Engng. Struct. Dyn.* **27,** 693–685 (1998).
22. L. E. Suarez and M. P.Singh, 'Dynamic synthesis of nonclassically damped substructures', *J. Engng. Mech. ASCE* **115** (1), 52–70 (1989).
23. A. Paronesso and J. P. Wolf, 'Global lumped-parameter model with physical representation for unbounded medium, *Earthquake Engng. Struct. Dyn.* **24,** 637–654 (1995).
24. J. H. Gordis, R. L. Bielawa and W. G. Flannelly, 'A general theory for frequency domain structural synthesis', *J. Sound Vib.* **150**(1), 139–158 (1991).
25. J. H. Gordis, 'Structural synthesis in the frequency domain: a general formulation', *Shock Vib.* **1** (5), 461–471, (1994).
26. R. H. MacNeal, 'A hybrid method of component-mode synthesis', *Comput. Struct.* **1,** 581–601, (1971).
27. P. D. Spanos and A. Majed, 'A residual flexibility approach for decoupled analysis of systems of combined components', *J. Vib. Acoust.* ASME **118,** 682–686 (1996).
28. A. Majed and P. D. Spanos, 'Nonlinear dynamics of structures via residual flexibility of components', *J. Aerospace Engng.* ASCE, **10(**4) (1997).
29. G. H. Golub and C. F. Van Loan, *Matrix Computations*, 2nd edn, the Johns Hopkins University Press, Baltimore, 1989.
30. E. H. Vanmarcke, 'Structural response to earthquakes, in C. Lomitz and E. Rosenblueth (eds.), *Seismic Risk and Engineering Decisions*, Holland, Elsevier, Amsterdam, 1976.
31. M. B. Priestley, 'Evolutionary spectra and non-stationary processes', *J. Roy. Statis. Soc.* Series B **27,** 204–237 (1965).
32. Y. K. Lin and G. Q. Cai, *Probabilistic Structural Dynamics*: *Advanced Theory and Applications*, McGraw-Hill, New York, 1995.
33. M. K. Kaul, 'Stochastic characterization of earthquakes through their response spectrum', *Earthquake Engng. Struct. Dyn.* **6,** 497–509 (1978).
34. A. Y. Leung, 'An accurate method of dynamic condensation in structural analysis', *Int. J. Numer. Meth. Engng.* **12,** 1705–1715 (1978).
35. J. P. Wolf, 'Spring-dashpot-mass models for foundation vibrations', *Earthquake Engng. Struct. Dyn.* **26**, 931–949 (1997).
36. Y. Yang and W. Huang, 'Seismic response of light equipment in torsional buildings', *Earthquake Engng. Struct. Dyn.* **22,** 113–128 (1993).

